

# Reply to the Comment on “Spin and orbital angular momentum in gauge theories: Nucleon spin structure and multipole radiation revisited” [PRL 100:232002 (2008)]

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As the comment of Tiwari [1] reflects typical and probably common misunderstandings, we feel it valuable to elaborate further on these delicate and important issues:

(i) In the gauge coupling  $\bar{\psi}\gamma^\mu A_\mu\psi$ , the gauge field  $A_\mu$  play a dual role: it provides a physical coupling to the Dirac field  $\psi$ , as well as a gauge freedom to compensate for the phase freedom of  $\psi$ . Our idea of solving the gauge-invariance problem is to decompose this dual role by seeking, in any gauge, a unique separation  $\vec{A} = \vec{A}_{\text{pure}} + \vec{A}_{\text{phys}}$ , with  $\vec{A}_{\text{pure}}$  a pure-gauge term transforming in the same manner as does the full  $\vec{A}$  and always giving null  $\vec{B}$ , and  $\vec{A}_{\text{phys}}$  a physical term transforming in the same manner as does the electric field  $\vec{E}$ . Namely,  $\vec{A}_{\text{phys}}$  is gauge invariant/covariant in electrodynamics/Yang-Mills theory, while  $\vec{A}_{\text{pure}}$  has the same gauge freedom as  $\vec{A}$  and can be used instead of  $\vec{A}$  to construct a covariant derivative  $\vec{D}_{\text{pure}} \equiv \vec{\nabla} - i\vec{A}_{\text{pure}}$ . The separation  $\vec{A} = \vec{A}_{\text{pure}} + \vec{A}_{\text{phys}}$ , and the definitions for  $\vec{A}_{\text{phys}}$  and  $\vec{A}_{\text{pure}}$ , are by no means specifying a gauge or restricting the gauge freedom of  $\vec{A}$ . E.g., in electrodynamics,  $\vec{\nabla} \cdot \vec{A}_{\text{phys}} = 0$  is the definition of  $\vec{A}_{\text{phys}}$  in any gauge, it is not to be confused with the gauge condition  $\vec{\nabla} \cdot \vec{A} = 0$ . Nevertheless, the separation  $\vec{A} = \vec{A}_{\text{pure}} + \vec{A}_{\text{phys}}$  does become the simplest in a unique physical gauge in which  $\vec{A}_{\text{pure}} = 0$  and  $\vec{A}_{\text{phys}} = \vec{A}$ . Coulomb gauge  $\vec{\nabla} \cdot \vec{A} = 0$  is the physical gauge for electrodynamics. But for Yang-Mills theory the physical gauge is  $[\vec{A}, \vec{E}] = 0$  which makes the gauge-dependent gluon color charge vanish [2], while  $\vec{\nabla} \cdot \vec{A} = 0$  is no longer privileged. This may elucidate why quantization of Yang-Mills theory in the Coulomb gauge is not very illuminating.

(ii) Our solution achieves the best possible reconciliation of Lorentz covariance and gauge invariance for the momentum and angular momentum (AM) of coupled quarks and gluons. To appreciate this point, we first note that, in the instant form, six Poincaré generators can be interaction-free (or *good*):  $\vec{P} = \vec{P}_q + \vec{P}_g$ ,  $\vec{J} = \vec{J}_q + \vec{J}_g$ , and four generators involve interactions:  $H = H_q + H_g + H_{\text{int}}$ ,  $\vec{K} = \vec{K}_q + \vec{K}_g + \vec{K}_{\text{int}}$ . Here  $q$ ,  $g$ , and  $\text{int}$  denote the quark, gluon, and quark-gluon interacting parts, respectively. In gauge theories, it should be further noted that not only the interaction-involving Lorentz transformations, but also the interaction-free ones, are

troublesome. This is because an interaction-free operator, like  $\int d^3x \psi^\dagger \vec{x} \times \frac{1}{i} \vec{\nabla} \psi$ , is often gauge-dependent, while a gauge-invariant construction, like  $\int d^3x \psi^\dagger \vec{x} \times \frac{1}{i} \vec{D} \psi$ , often involves interaction. In fact, the essence of our contribution is that the *good* generators  $\vec{P}$  and  $\vec{J}$  are indeed constructed to be both interaction-free and gauge independent, so that each part in  $\vec{P}$  and  $\vec{J}$  transforms properly under spatial translations and rotations. But since  $\vec{K}$  involve interaction intrinsically, only the *total*  $\vec{J}$  transform properly under boost:  $[K^i, J^j] = i\epsilon_{ijk}K^k$ , while  $[K^i, J_{q,g}^j] = i\epsilon_{ijk}K_{q,g}^k$  can *not* hold (otherwise we would have  $[K^i, J_q^j + J_g^j] = i\epsilon_{ijk}[K_q^k + K_g^k]$ , which contradicts  $[K^i, J^j] = i\epsilon_{ijk}K^k$ ). To determine the boost properties of  $\vec{J}_{q,g}$ , we need to carry out a canonical quantization (preferably in the physical gauge so that the pure-gauge term vanishes), then compute the commutators  $[K^i, J_{q,g}^j]$ . This task is difficult but unavoidable for everyone. In comparison, the light-cone formalism renders boost along the third axis interaction-free, but makes the description of AM rather troublesome, because two AM components (namely, the rotation generators along the  $x$ ,  $y$  axes) involve interactions. All in all, the separation of  $\vec{P}$  or  $\vec{J}$  into quark and gluon parts can never be fully covariant under all Lorentz transformations, no matter how one deals with the gauge field. In fact, this property is an intrinsic complication for any interacting system, not merely gauge interactions. By decomposing  $\vec{A}$  into physical and pure-gauge terms, we add absolutely no extra complication in this regard.

(iii) It is an illusion that the spin and orbital AM of light as in the optic measurements or multipole-radiation analysis can be straightforwardly interpreted in classical electrodynamics, without applying the approach which we follow. As long as one discusses spin and orbital AM separately, the gauge-invariance problem is sharply encountered, no matter one adopts a classical or quantum language. This problem is just absent if one merely concerns about the integrated total AM. Without defining the spin and orbital AM gauge-invariantly, it is impossible to interpret the optic measurements which intend to manipulate spin and orbital AM separately, or the multipole-radiation analysis which employs the notion of spin-orbital coupling. We should also clarify that the solution we provide applies to both classical and quantum

dynamics. It is our gauge-invariant construction for the spin and orbital AM that justifies the spin and orbital values or quantum numbers assigned to the electromagnetic field or photon in any experiment. And in turn, the consistency of our theory with experimental results supports the validity and correctness of our gauge-invariant expressions.

(iv) To see the physical content of  $\vec{E} \times \vec{B}$ , note that  $\vec{J} = \int d^3x \vec{x} \times (\vec{E} \times \vec{B})$  gives the total AM of a free gauge field, including both spin and orbital parts. But if  $\vec{P}(x)$  is the momentum density, then  $\int d^3x \vec{x} \times \vec{P}(x)$  is the standard form of orbital AM. This implies that  $\vec{E} \times \vec{B}$  is not a pure mechanical momentum, it must include a spin flow so as to give the total  $\vec{J}$  by an apparent orbital form. By writing  $\vec{J} = \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times (E^i \vec{\nabla} A_{\text{phys}}^i)$ , the correct momentum density is read out to be  $E^i \vec{\nabla} A_{\text{phys}}^i$ . From  $\vec{E} \times \vec{B} = \vec{E}^i \vec{\nabla} A_{\text{phys}}^i - \nabla^i (E^i \vec{A}_{\text{phys}}) + (\vec{\nabla} \cdot \vec{E}) \vec{A}_{\text{phys}}$ , we see that the difference between  $\vec{E} \times \vec{B}$  and  $E^i \vec{\nabla} A_{\text{phys}}^i$  is a surface term only if the (in general charged, here) Dirac field is absent so that  $\vec{\nabla} \cdot \vec{E} = 0$ . For an interacting system, the difference is substantial. Similar situation occurs for the AM:  $\vec{x} \times (\vec{E} \times \vec{B}) = \vec{E} \times \vec{A}_{\text{phys}} + \vec{x} \times (E^i \vec{\nabla} A_{\text{phys}}^i) + \nabla^i [E^i (\vec{A}_{\text{phys}} \times \vec{x})] - (\vec{\nabla} \cdot \vec{E}) \vec{A}_{\text{phys}} \times \vec{x}$ , hence  $\int d^3x \vec{x} \times (\vec{E} \times \vec{B})$  may differ drastically from  $\int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times (E^i \vec{\nabla} A_{\text{phys}}^i)$  in the presence of interaction. For the nucleon, then,  $\vec{E} \times \vec{B}$  and  $E^i \vec{\nabla} A_{\text{phys}}^i$

may give totally different views of how much nucleon momentum is carried by gluons.  $\vec{E} \times \vec{B}$  is a component of the symmetric energy-momentum tensor, while  $E^i \vec{\nabla} A_{\text{phys}}^i$  belongs to the (non-symmetric) canonical one. These two tensors differ by a total-derivative spin term. This term does not matter for the integrated total energy or momentum, so there is no curiosity in that a photon with arbitrary polarization and orbital AM can have the same energy. However, the spin term does alter the density and symmetry of the energy-momentum tensor, for which we do need a concrete density expression for the purpose of coupling to gravity. Thus, were one to adopt Einstein's gravitational equation which requires a symmetric tensor, then the "momentum" that contributes to gravity is not the mechanical momentum [3].

In closing, we remark that Ref. [2] does solve the long-standing gauge-invariance problem of spin and orbital AM, in the sense that the expressions satisfy all theoretical requirements and agree with all experimental results.

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- [1] S.C. Tiwari, arXiv:0807.0699.
  - [2] X.S. Chen, X.F. Lü, W.M. Sun, F. Wang, and T. Goldman, Phys. Rev. Lett. **100**, 232002 (2008).
  - [3] X.S. Chen *et al*, arXiv:0710.1427 [hep-ph].